ABSTRACT

Safety assessment of facilities involved in geological disposal of hazardous waste, including radioactive nuclear waste, is generally performed through mass transport simulations combined with uncertainty and sensitivity analyses. Transport of contaminants, such as radionuclides, through an engineered and/or natural barrier system is mainly controlled by advection, dispersion, sorption, and chain decay. Ideally, waste disposal facilities should be constructed in the geological environments where groundwater is not existent, or groundwater is static, or its flow is extremely slow. Potential fluid flow, however, may be induced by thermal convection and/or gas generation, and thus accurate evaluation of hydraulic properties, specifically the permeability and specific storage, along with diffusive transport properties of engineered and natural barrier materials, is of fundamental importance for safety assessment. The engineered and natural barrier materials for isolating hazardous wastes are hydraulically tight, and special techniques are generally required to obtain both rapid and accurate determination of their hydraulic properties. In this paper, the constant flow technique is introduced and evaluated. The capability of this technique in testing low-permeability geo-materials are illustrated through practical applications to a bentonite-sand mixture and rock samples having low permeabilities.

INTRODUCTION

Safety assessment of facilities involved in geological disposal of hazardous waste, including radioactive nuclear waste, is generally performed through mass transport simulations combined with uncertainty and sensitivity analyses. Transport of contaminants, such as radionuclides, through an engineered and/or natural barrier system is mainly controlled by advection, dispersion, sorption, and chain decay. Ideally, waste disposal facilities should be constructed in the geological environments where groundwater is not existent, or groundwater is static, or its flow is extremely slow. Potential fluid flow, however, may be induced by thermal convection and/or gas generation, and thus accurate evaluation of hydraulic together with diffusive transport properties of engineered and natural barrier materials is of fundamental importance for safety assessment.

The diffusive transport properties, specifically the effective diffusion coefficient and sorption coefficient or rock capacity factor, are generally characterized by using laboratory diffusion tests as well as batch experiments, and numerous valuable literatures are available on such research subjects [e.g., 1-14]. Recently, a systematic evaluation of potential problems associated with conventional laboratory diffusion and batch experiments, and perspective improvements to the laboratory diffusion tests has also been performing by the authors [15-19].

The hydraulic properties, specifically the permeability and specific storage, of geo-materials are generally determined through laboratory permeability tests. The accurate and reliable determination of low permeabilities in the laboratory, however, is typically involved and time consuming. Conventional laboratory test methods such as the constant-head and falling-head techniques estimate the permeability
of a saturated specimen by measuring induced flow rates [e.g. 20]. Because common fluid-volume measurement devices have a practical resolution limitation on the order of $10^{-3}$ml, measurements on low permeability geo-materials can be achieved only if the tests are monitored for a very long time or if the imposed hydraulic gradients are very high. Both the prolonged test period and the higher hydraulic gradients will often induce significant errors in the permeability estimations. The former may reflect errors caused by temperature fluctuations and bacterial growth, whereas the latter may produce sample disturbance resulting from seepage-induced consolidation for softer samples like clay soils as well as non-uniform distribution of effective stress within the test specimen. To obtain representative measurements of permeability in the laboratory, test conditions, typically the confining and pore pressures, and the hydraulic gradient, should be controlled as close as possible to those existing in situ.

In this paper, the constant flow technique capable of testing low permeability geo-materials is introduced and recent improvements to the analytical solution for this technique is overviewed. Evaluation of the constant flow technique is performed through theoretical analyses and simulations, and the capability of this technique is illustrated through practical applications of this technique to test a bentonite-sand mixture and rock samples having low permeabilities.

**CONCEPT AND THEORY OF CONSTANT FLOW TECHNIQUE**

**Basic Concept**

When testing low-permeability specimens, it is difficult to maintain a constant hydraulic pressure over a long time and to measure precisely the small flow rates that ensue. The constant flow technique relies on the opposite strategy of maintaining a constant, small flow rate and measuring the corresponding pressure response. The permeability of the specimen is proportional to the steady-state differential pressure held across it and is determined by Darcy’s law for flow through saturated porous media [21]. The concept, or schematic diagram, as well as the boundary conditions, associated with a constant flow test arrangement can be depicted as shown in Fig. 1 [22].

![Fig. 1 Schematic diagram and the boundary conditions associated with the constant flow test arrangement.](image-url)

Although several decades have passed since Olsen [21] proposed this technique (formerly named as flow-pump method), this method has not been widely used in the laboratory. This may partially due to higher equipment costs with a few qualitatively described benefits. In addition, rigorous solution to this
test method was not developed until 1996 [22, 23]. A brief introduction to the mathematical model and analytical solutions to this test method is illustrated in the following sessions.

**Mathematical Model**

The mathematical model of a constant flow test can be expressed as follows:

**Governing equation**

\[
\frac{\partial^2 H}{\partial z^2} - \frac{S_s}{K} \cdot \frac{\partial H}{\partial t} = 0
\]  

(1)

**Initial condition**

\[ H(z,0) = 0 \quad 0 \leq z \leq L \]  

(2)

**Boundary condition**

\[ H(0,t) = 0 \quad t \geq 0 \]  

(3)

\[ \frac{\partial H(L,t)}{\partial z} = \frac{Q(t)}{K \cdot A} \quad t > 0 \]  

(4)

In which

\[ Q(t) = q - C_e \cdot \frac{\partial H(L,t)}{\partial t} \]  

(5)

Eq. (5) means that the actual flow rate entering the upstream end of test specimen at time \( t \) equals the constant flow rate generated from the flow pump minus the volume absorbed within the upstream permeating system per unit time interval.

Here, \( H \) is the hydraulic head, \( z \) is the vertical distance along the specimen, \( S_s \) is the specific storage of the specimen, \( K \) is the permeability of the specimen, \( t \) is the time from the start of the experiment, \( L \) is the length of the specimen, \( A \) is cross-sectional area of specimen, \( Q(t) \) is flow rate into the upstream end of specimen at time \( t \), \( q \) is constant flow rate generated by flow pump, and \( C_e \) is storage capacity of the upstream permeating system, i.e., the change in volume of the permeating fluid in the upstream permeating system per unit change in hydraulic head.

**Analytical Solution and Discussion**

The solution to the above mathematical model, i.e., the rigorous solution to the constant flow test, can be obtained by the Laplace transform method and expressed as follows:

\[
H(z,t) = \frac{q \cdot L}{A \cdot K} \cdot \left\{ \frac{\exp\left(-\frac{K}{S_s} \cdot \beta_n^2 \cdot t\right) \cdot \sin(\beta_n \cdot z)}{L \cdot \delta \cdot \beta_n \cdot \cos(\beta_n \cdot L) \cdot \left[L \cdot \left(\beta_n^2 + \frac{1}{\delta^2}\right) + \frac{1}{\delta}\right]} \right\}
\]  

(6)

In which \( \delta = \frac{C_e}{(A \cdot S_s)} \), and \( \beta_n \) are the roots of following equation

\[
\tan(\beta \cdot L) = \frac{1}{\beta \cdot \delta}
\]  

(7)

In the application of interest, the differential head \( H \) is measured across the entire length of the specimen. Therefore, we allow \( z=L \) and Eq. (6) reduces to the following expression, noticing that \( \tan \beta \) satisfies Eq. (7):
The time-imposed initial condition of the experimental method is satisfied since \( H=0 \) at \( t=0 \). At large times, Eq. (8) simplifies to the steady state solution defined by Darcy’s law.

When we disregard the storage capacity of the upstream permeating system, i.e., let \( C_s = 0 \), and correspondingly, \( \delta = 0 \). The roots of Eq. (7) become \((2n+1)\pi/(2L)\) and Eq. (8) reduces to the following expression as developed by Morin and Olsen in 1987 [24]:

\[
H = \frac{q \cdot L}{A \cdot K} \left\{ 1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{\exp\left(-\frac{K}{S_s} \cdot (2n+1)^2 \cdot \pi^2 \cdot t / (4L^2)\right)}{(2n+1)^2} \right\} \tag{9}
\]

Note that Eq. (9) can not be effectively used to test very low permeability because the storage capacity of the upstream permeating system is not considered, and the specific storage derived from Eq. (9) may include large errors.

**SIMULATION AND THEORETICAL EVALUATION**

To discuss the characteristics of the constant flow technique, the hydraulic gradient distribution within the specimen during a constant flow permeability test can be further derived by differentiating the Eq. (6) with respect to the variable \( z \):

\[
i(z,t) = \frac{q \cdot L}{A \cdot K} \left\{ 1 - \frac{2}{L} \sum_{n=0}^{\infty} \frac{\exp\left(-\frac{K}{S_s} \cdot \beta_n^2 \cdot t \cdot \cos(\beta_n \cdot z)\right)}{L \cdot \delta \cdot \cos(\beta_n \cdot L) \cdot \left[ L \cdot \left( \beta_n^2 + \frac{1}{\delta^2} \right) + \frac{1}{\delta} \right]} \right\} \tag{10}
\]

Using Eqs. (6), (7) and (10), and by replacing the variables with pertinent values, the particular response characteristics of hydraulic head and hydraulic gradient within test specimen can be simulated and evaluated. The permeability of a geo-material depends upon its porosity and the connectivity between pores. Its specific storage is a function of the interconnected porosity, the bulk and matrix compressibilities, and also the compressibility of the saturating pore fluid. Accordingly, the hydrologic properties of geo-materials may vary dramatically. In the following simulations, parameter values (as shown in the figures) are cited from the experimental results obtained by Zhang et al. [22, 25].

The time-dependent changes in hydraulic head and hydraulic gradient within the specimen during the constant flow permeability test are illustrated in Figs. 2 and 3, respectively. Even using the constant flow technique, the time required to establish the steady state is still relatively long, up to several tens of hours in the simulated case, for testing low permeability geo-materials. Examination of Eq. (6) shows that the lower the permeability, and/or the larger the specific storage of the specimen, and/or the larger the storage capacity of upstream permeating system, the longer will be the time required to establish steady state. Transient conditions will also persist for a longer time if the specimen length, \( L \), is increased. In other words, to obtain steady state as early as possible, it is advisable to use test specimens with pertinently short lengths.
At the onset of the experiment, the increase in hydraulic head at the base of the specimen is relatively gradual (Fig. 2), and correspondingly, the hydraulic gradients are established slowly across the entire specimen until a steady state bound is reached that is also the maximum hydraulic gradient value generated during the test. An examination of Eq. 10 also finds that the maximum hydraulic gradient is proportional to the value of the constant flow rate to be set during a test. In other words, the maximum hydraulic gradient during a test can be controlled by properly setting the small value of flow rate for the test. This feature of the constant flow technique facilitates simulating low hydraulic gradients, and thus laminar flow as existing in natural environments.
APPLICATION EXAMPLES

Application to Testing A Bentonite-Sand Mixture

Fig. 4 shows the schematic drawing of a constant flow permeability test system which was developed by the first author to investigate the influence of shear strain on the hydraulic properties of a bentonite-sand mixture to be used in a final land disposal facility of low-level radioactive nuclear waste in Japan. Shear was imposed by means of the differential pressure between the external and internal pressures [22, 25].

![Fig. 4 Schematic drawing of coupled shear and permeability test system with constant flow technique for testing bentonite-sand mixture.](image)

A permeability test was conducted at a flow rate of $8.5 \times 10^{-12} \text{ m}^3 / \text{s}$ on a thick-walled cylindrical specimen consisting of a compacted bentonite-sand mixture, measuring 10 cm and 4 cm in outer and inner diameters by 4 cm in length. Taking the result obtained under the external pressure was 0.5 MPa and the internal pressure is 0.1 MPa as an example, the transient hydraulic head across the specimen measured versus time during the permeability test is plotted in Fig. 5.

Calculating the values for the permeability and specific storage of test specimen, as well as the storage capacity of upstream permeating system from experimental results were performed by means of the parameter identification technique, a technique widely used in the field of system engineering [22, 23, 25, 26]. Initial values and searching areas for the permeability, the specific storage of the specimen and the storage capacity of upstream permeating system, and their values determined from the laboratory measurements, are tabulated in Table I. The simulated curve using the rigorous solution (Eq. 8) and the parameters identified from the entire test time, and the predicted curve using the parameters identified from the first 15 hours are also shown in Fig. 5.

The accuracy and effectiveness of the analysis, correspondingly the applicability of constant flow technique with the rigorous analytical theory, are demonstrated by the following observations: 1) the permeability obtained from the early time measurements is almost the same as that obtained from the entire test time, and that obtained from the steady state using Darcy’s law, 2) the storage capacities of the
specimen and of the upstream permeating system show negligible change across different test periods, and 3) the simulated and the predicted curves are in close agreement with the laboratory data that means the early time evaluation of the parameter is possible with a sufficient precision.

The results of the effects of shear strain on hydraulic properties of the bentonite-sand mixture are given in the literature [25] and beyond this paper.

![Fig. 5 Comparison of the measured, simulated and predicted hydraulic head versus time.](image)

Table I Initial values, searching areas and identified values of the parameters associated with the constant flow permeability test

<table>
<thead>
<tr>
<th></th>
<th>Permeability (m/s)</th>
<th>Specific storage (1/m)</th>
<th>Storage capacity of upstream permeating system (m³/MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Searching area</td>
<td>1E-11-1E-10</td>
<td>1E-4-1E-2</td>
<td>1E-7-1E-5</td>
</tr>
<tr>
<td>Initial value</td>
<td>----</td>
<td>1E-3</td>
<td>1E-6</td>
</tr>
<tr>
<td>Early time identification 1</td>
<td>6.62E-11</td>
<td>3.15E-3</td>
<td>3.01E-6</td>
</tr>
<tr>
<td>Early time identification 2</td>
<td>6.57E-11</td>
<td>3.29E-3</td>
<td>2.89E-6</td>
</tr>
<tr>
<td>Whole test time identification</td>
<td>6.60E-11</td>
<td>3.29E-3</td>
<td>2.75E-6</td>
</tr>
<tr>
<td>Steady-state analysis</td>
<td>6.69E-11</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>

Note: Early time 1 and 2 means the first 15 and 25 hours of the entire 43 hours of the test duration, respectively.

**Application to Testing Rock Samples**

Fig. 6 shows the schematic of a recently-developed versatile laboratory permeability system capable of testing rock samples either by constant head, constant flow or pulse technique. Details of the system components, their technical features and test procedures are provided in literature [26]. The results
derived from two types of rock that are commonly available in Japan by using the constant flow technique are introduced to illustrate the capability of the test method.

Inada granite was used as a typical example of a crystalline rock, with low porosity (less than 0.6%) and little compressive storage. Shirahama sandstone was used as an example of a sedimentary rock, having relatively large porosity (larger than 13%) and thus relatively large compressive storage. The sample of Inada granite was cored in the direction perpendicular to Hardway plane and the sample of Shirahama sandstone was cored perpendicular to bedding. To shorten the time required for a test, disc-shaped cylindrical specimens having dimensions of 50 mm in diameter by 25 mm in length were used. In testing the Shirahama sandstone, pore pressure was kept at 1 MPa and the confining pressure was increased stepwise from 2, 4 to 10 MPa, and the constant flow rates were controlled at 0.005, 0.002 and 0.001 cm³/min, respectively. In testing the Inada granite, pore pressure was first set to be 1 MPa and the confining pressure was set to be 2 MP. The constant flow rate was controlled at 0.0005 cm³/min. The pore pressure was then increased to 2 MP and the confining pressure was then increased stepwise from 10 MPa to 20 MPa, and the constant flow rate was controlled at 0.0003 cm³/min.

Similar to the results obtained from testing the bentonite-sand mixture, the data obtained from the constant flow permeability tests depict the time dependent rise in differential hydraulic head across the specimen and are shown in Fig. 7. For comparison, simulated curves are also provided for individual tests. The permeability and specific storage of each specimen under different test conditions are calculated from the data obtained over the whole-test and half-test times, again by means of the parameter identification technique, and are tabulated in Table II. Permeability values derived from the steady-state measurements are also listed in the same table.

The simulated curves are in close agreement with the experimental data. These results demonstrate again the accuracy of the rigorous theoretical analyses for individual tests. By properly setting the small levels for the constant flow rate, relatively low hydraulic gradients, less than about 30 in the experimental examples (the maximum differential hydraulic head across the specimen/specimen length), can be controlled in constant flow permeability tests. The values for both the permeability and specific storage identified from the half-time experimental data are in close agreement with those identified from the
whole-time experimental data. These permeability values are also similar to those calculated from the steady-state measurements. These results indicate again that using the rigorous theoretical analysis together with the parameter identification technique not only reduces the time required for testing low-permeability specimens, but also permits the specific storage to be determined as well.

Table II Results obtained from constant flow permeability tests on Shirahama sandstone and Inada granite.

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Test No.</th>
<th>$K_h$ (cm/s)</th>
<th>$K_w$ (cm/s)</th>
<th>$K_s$ (1/cm)</th>
<th>$S_{sh}$ (1/cm)</th>
<th>$S_{sw}$ (1/cm)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shirahama sandstone</td>
<td>SVS21FP</td>
<td>1.04E-7</td>
<td>1.04E-7</td>
<td>1.04E-7</td>
<td>1.40E-6</td>
<td>1.41E-6</td>
<td>510</td>
</tr>
<tr>
<td></td>
<td>SVS41FP</td>
<td>0.51E-7</td>
<td>0.51E-7</td>
<td>0.51E-7</td>
<td>2.51E-6</td>
<td>2.62E-6</td>
<td>1930</td>
</tr>
<tr>
<td></td>
<td>SVS101FP</td>
<td>0.40E-7</td>
<td>0.43E-7</td>
<td>0.43E-7</td>
<td>2.76E-6</td>
<td>2.62E-6</td>
<td>1580</td>
</tr>
<tr>
<td>Inada granite</td>
<td>GHL21FP</td>
<td>4.28E-9</td>
<td>4.77E-9</td>
<td>4.85E-9</td>
<td>3.60E-7</td>
<td>6.13E-7</td>
<td>1980</td>
</tr>
<tr>
<td></td>
<td>GHL102FP</td>
<td>2.23E-9</td>
<td>2.40E-9</td>
<td>2.43E-9</td>
<td>6.00E-7</td>
<td>4.03E-7</td>
<td>5280</td>
</tr>
<tr>
<td></td>
<td>GHL202FP</td>
<td>1.71E-9</td>
<td>1.76E-9</td>
<td>1.75E-9</td>
<td>3.08E-7</td>
<td>2.37E-7</td>
<td>6360</td>
</tr>
</tbody>
</table>

Note: Subscripts h, w and s mean the estimations from the half-time, whole-time and steady-state experimental data, respectively. All the values of permeability are converted at the temperature of 15°C.

CONCLUSIONS

Safety assessment of facilities involved in geological disposal of hazardous waste, including radioactive nuclear waste, is generally performed through mass transport simulations combined with uncertainty and sensitivity analyses. Transport of contaminants, such as radionuclides, through an engineered and/or natural barrier system is mainly controlled by advection, dispersion, sorption, and chain decay, and thus pertinent determination of relevant parameters is of fundamental importance for safety assessment. In this paper, the constant flow technique capable of testing low permeability geo-materials is introduced. Evaluation of the technique is performed with theoretical analyses, and the applicability of
this technique is demonstrated by experimental examples on testing a bentonite-sand mixture and two different rock samples. Primary conclusions drawn from this paper are as follows:

1) The maximum hydraulic gradient within test specimen during a constant flow permeability test can be controlled by appropriately setting the small value of constant flow rate.

2) The rigorous solution to the constant flow permeability test can be used to interpret precisely the experimental data. The use of rigorous solution together with the parameter identification technique permits test duration to be significantly shortened without sacrificing accuracy in estimating both the permeability and specific storage of a test specimen.

3) With meticulously-designed laboratory systems, the constant flow technique can be used to test low permeability geo-materials up to high confining and pore pressure conditions, and with relatively low hydraulic gradients.

REFERENCES


